1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.

Answer :

**Probability That All 4 Seasons Occur at Least Once Among 7 Birthdays**

This is a problem of finding the probability that in a group of 7 people, the birthdays are spread across all four seasons at least once.

**Solution Approach:**

We can use the **inclusion-exclusion principle** to solve this:

* There are 474^747 total possible ways to assign the seasons to the 7 people (since each person can have one of the 4 seasons for their birthday).
* The number of ways in which one or more seasons are missing can be calculated and subtracted from the total.

The probability PPP that all 4 seasons occur at least once is:

P=1−(37−(42)×27+(43)×1747)P = 1 - \left(\frac{3^7 - \binom{4}{2} \times 2^7 + \binom{4}{3} \times 1^7}{4^7}\right)P=1−(4737−(24​)×27+(34​)×17​)

Calculating this directly involves computing the specific values, but for simplicity, you can use:

P=1−37−6×27+4×1747P = 1 - \frac{3^7 - 6 \times 2^7 + 4 \times 1^7}{4^7}P=1−4737−6×27+4×17​

This provides the exact probability.

1. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?

Alice is choosing 7 classes out of 30, with 6 classes available per day. We need to find the probability that she ends up with at least one class on each weekday.

**Solution Approach:**

1. **Total Ways to Choose 7 Classes:**

Total possible combinations=(307)\text{Total possible combinations} = \binom{30}{7}Total possible combinations=(730​)

1. **Favorable Outcomes (At Least One Class Per Day):**

First, we find the total number of ways to choose classes such that there is at least one class each day. This is done by ensuring she has 1 class on each of the 5 days, and then choosing 2 additional classes from the remaining 25.

Favorable combinations=(61)5×(252)\text{Favorable combinations} = \binom{6}{1}^5 \times \binom{25}{2}Favorable combinations=(16​)5×(225​)

1. **Probability:**

P=(61)5×(252)(307)P = \frac{\binom{6}{1}^5 \times \binom{25}{2}}{\binom{30}{7}}P=(730​)(16​)5×(225​)​

This gives the probability that Alice will have classes every day from Monday through Friday.